DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

Econometrics 2, Class 1

Problem Set #7
October 31, 2005

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

Problem Set #7

- We start with a multiple choice test.
- You get the rest of the hour to answer the questions (in groups if you wish).
- Give reasons for your answers.
- Again, you are welcome to present your answers!

7.1 Questionnaire 1-5

Table 7.1: Questionnaire		(a)	(b)
1	Consider the AR(2) process, $Y_t = \delta + \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \epsilon_t$. When is the maximum likelihood estimator identical to the OLS estimator? (a) when $\epsilon_t \sim N(0, \sigma^2)$; (b) always.		
2	Consider the AR(1) process, $Y_t = 5 + 0.8 \cdot Y_{t-1} + \epsilon_t$. What is the mean, $E[Y_t]$? (a) $E[Y_t] = 5$; (b) $E[Y_t] = 25$.		
3	Minimizing an information criteria, e.g. AIC, HQ or SC, will give the same result as maximizing R^2. (a) correct; (b) wrong.		
4	Is a finite moving average process always stationary? (a) yes; (b) no.		
5	Can the moving average model be estimated by OLS? (a) yes; (b) no.		

Questionnaire 6-10

6	For a stationary process, the autocorrelations, $\rho_1, \rho_2, ...$, are always zero? (a) yes; (b) no.		
7	Chooce the correct orders for the following ARIMA model for Y_t: $\Delta Y_t = \delta + \theta_1 \Delta Y_{t-1} + \epsilon_t + \alpha_1 \epsilon_{t-1} + \alpha_2 \epsilon_{t-2}$. (a) ARIMA(1,1,2); (b) ARIMA(2,0,1).		
8	Why are unit roots important in time series analysis? (a) the usual asymptotic results do not hold; (b) it is important for forecasting.		
9	For an ARMA(2,0) model which one is correct: (a) the two first ACFs are significant while all PACFs are insignificant; (b) the ACF converges exponential to zero while the first two PACFs are significant.		
10	Consider the model $y_t = \theta y_{t-1} + \epsilon_t + \alpha \epsilon_{t-1}$. The optimal forecast for $y_{T+3\|T}$ is: (a) $y_{T+3\|T} = \widehat{\theta}^3 y_T + \widehat{\theta}^2 \widehat{\alpha} \widehat{\epsilon}_T$; (b) $y_{T+3\|T} = \widehat{\theta}^3 y_T$.		

7.2 ARIMA Models for Danish Time Series

We have quarterly observations of the following variables over the last approximately 30 years:

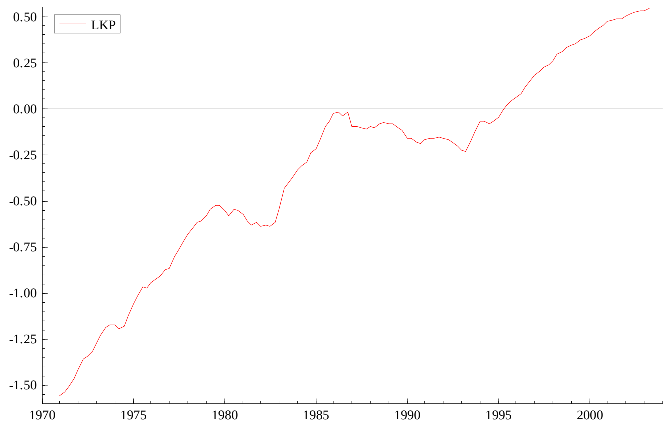
EFKRKS	Effective (nominal) exchange rate (1980=100).
ENL	Current account (current prices).
FIPMXE	Business machinery investments (constant prices).
FY	Total GDP (constant prices).
IBZ	Bond yield (pro anno yield in ratio).
KP	House price index (1995=1).
PCP	Consumption deflator (1995=1).
UNR	Unemployment rate (in ratio).
TT	Terms of trade, export price/import price, (1995=1).

(1) Choose a time series for analysis

- I choose the house price index (*kp*)!
- I decide to transform this variable by taking the natural logarithm.
- I call the new variable *lkp*. (Use the Calculator in GiveWin.)
- Usually it is a good idea to transform all real variables (which don't cross zero!).
- This often helps to remove heteroskedasticity, where the residuals increase with the value of the variable. This happens because the error or change in the value of an outcome variable is often a **percent** of the value rather than an **absolute** value. For the same percent error, a bigger value of the variable means a bigger absolute error, so residuals are bigger too.
- log(Y*error) = log(Y) + log(error). The percent error therefore becomes the same additive error, regardless of the value of Y.

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

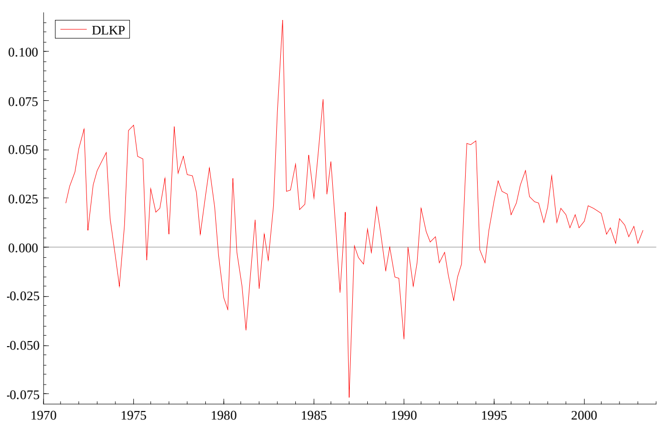
(2) Graph of the time series



This does not look very stationary!

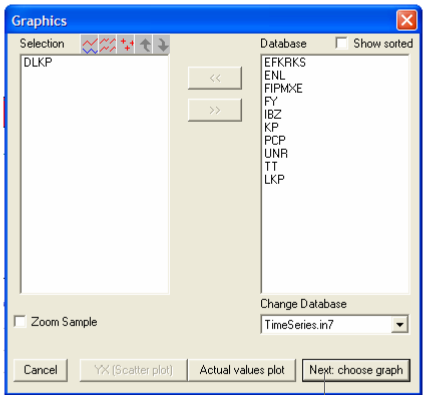
DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

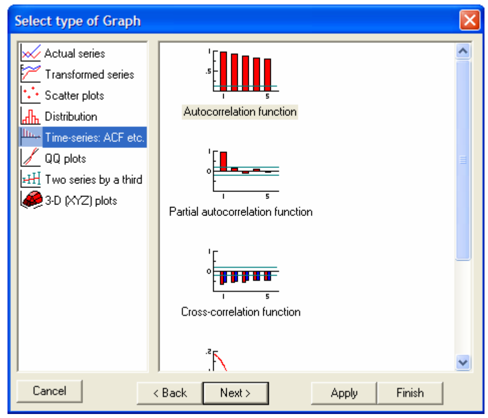
Graph of first differences



That looks much better!

(3) Draw the ACF and the PACF

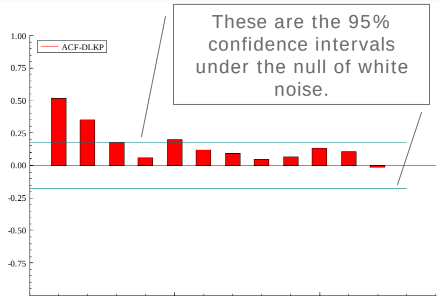




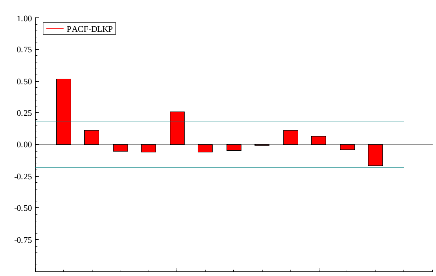
Go to the usual graphics menu, select the variable you want to analyze and click here.

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

ACF and PACF



These are the 95% confidence intervals under the null of white noise.



Remember: An AR(p) should have exponentially declining ACF and p significant PACF. An MA(q) should have q significant ACF and exponentially declining PACF.

This looks like an AR(2) process!

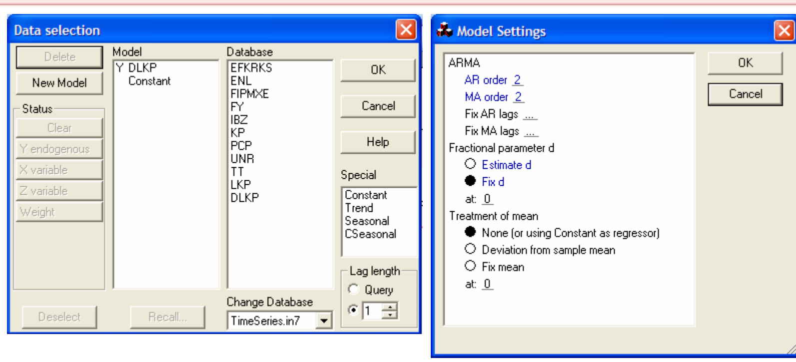
In the "old days" these graphs were used to decide which model to estimate (there wasn't time to estimate more than one!) Now these are often used to decide on the maximum number of lags to include.

(4) Estimate the relevant ARMA models

The relevant models are:

- ARMA(2,2)
- ARMA(2,1)
- ARMA(2,0)
- ARMA(1,2)
- ARMA(1,1)
- ARMA(1,0)
- ARMA(0,2)
- ARMA(0,1)
- ARMA(0,0)

GiveWin – Package – Time Series Models; Model – Formulate…



… and choose to estimate with maximum likelihood.

We start with the general model.

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

ARMA(2,2)

```
---- Maximum likelihood estimation of ARFIMA(2,0,2) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                   Coefficient  Std.Error  t-value  t-prob
AR-1                  0.436914     0.1534     2.85   0.005
AR-2                 -0.332963     0.2559    -1.30   0.196
MA-1               -0.00854203     0.1329  -0.0643   0.949
MA-2                  0.592704     0.2632     2.25   0.026
Constant             0.0163153   0.003418     4.77   0.000

log-likelihood     308.745105
no. of observations       129  no. of parameters          6
AIC.T              -605.490211  AIC               -4.69372256
mean(DLKP)           0.0163011  var(DLKP)         0.000689048
sigma                0.0220417  sigma^2           0.000485838

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
       1.0994      -0.39438       0.14397      -0.31449      0.016301
```

This is the fractional parameter, *d*, which we set equal to 0 (only relevant for ARFIMA models).

This is standard output. Already it looks like the data might best be described by an AR(1) process!

This is to do with how PcGive solved the maximum likelihood FOCs.

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

ARMA(2,1)

```
---- Maximum likelihood estimation of ARFIMA(2,0,1) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                   Coefficient  Std.Error  t-value  t-prob
AR-1                  0.311203     0.3735    0.833   0.406
AR-2                  0.188492     0.2006    0.939   0.349
MA-1                  0.141988     0.3721    0.382   0.703
Constant             0.0162866   0.004437     3.67   0.000

log-likelihood     307.198497
no. of observations       129  no. of parameters          5
AIC.T              -604.396993  AIC               -4.68524801
mean(DLKP)           0.0163011  var(DLKP)         0.000689048
sigma                0.0223349  sigma^2           0.000498848

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
      0.010966      0.34352       0.58537      0.016301
```

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

ARMA(2,0)

```
---- Maximum likelihood estimation of ARFIMA(2,0,0) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                  Coefficient  Std.Error  t-value  t-prob
AR-1                 0.452642    0.08697     5.20   0.000
AR-2                 0.114135    0.08698     1.31   0.192
Constant            0.0162902   0.004485     3.63   0.000

log-likelihood      307.138871
no. of observations        129  no. of parameters           4
AIC.T              -606.277742  AIC                -4.69982746
mean(DLKP)           0.0163011  var(DLKP)          0.000689048
sigma                0.0223454  sigma^2            0.000499317

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
      0.45623       0.11419      0.016301
```

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

ARMA(1,2)

```
---- Maximum likelihood estimation of ARFIMA(1,0,2) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                  Coefficient  Std.Error  t-value  t-prob
AR-1                 0.310253     0.1870     1.66   0.100
MA-1                 0.136802     0.1693    0.808   0.421
MA-2                 0.296248     0.1395     2.12   0.036
Constant            0.0162804   0.004025     4.05   0.000

log-likelihood      308.113642
no. of observations        129  no. of parameters           5
AIC.T              -606.227285  AIC                -4.69943631
mean(DLKP)           0.0163011  var(DLKP)          0.000689048
sigma                0.0221682  sigma^2             0.00049143

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
      0.51767       0.14440      -0.81381      0.016301
```

ARMA(1,1)

```
---- Maximum likelihood estimation of ARFIMA(1,0,1) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                    Coefficient  Std.Error  t-value  t-prob
AR-1                   0.642722     0.1195     5.38   0.000
MA-1                  -0.178087     0.1504    -1.18   0.239
Constant              0.0162970   0.004476     3.64   0.000

log-likelihood      306.973644
no. of observations        129  no. of parameters            4
AIC.T              -605.947288  AIC                 -4.6972658
mean(DLKP)           0.0163011  var(DLKP)          0.000689048
sigma                0.0223746  sigma^2            0.000500624

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
      0.67794     -0.32739      0.016301
```


ARMA(1,0)

```
---- Maximum likelihood estimation of ARFIMA(1,0,0) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                  Coefficient  Std.Error  t-value  t-prob
AR-1                 0.511624    0.07500     6.82   0.000
Constant            0.0162915   0.004022     4.05   0.000

log-likelihood      306.282782
no. of observations        129  no. of parameters            3
AIC.T              -606.565565  AIC                 -4.70205864
mean(DLKP)           0.0163011  var(DLKP)          0.000689048
sigma                0.0224964  sigma^2            0.000506088

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
      0.51504     0.016301
```

ARMA(0,2)

```
---- Maximum likelihood estimation of ARFIMA(0,0,2) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                     Coefficient  Std.Error  t-value  t-prob
MA-1                    0.379976    0.07905     4.81   0.000
MA-2                    0.381201    0.09196     4.15   0.000
Constant               0.0163014   0.003458     4.71   0.000

log-likelihood      306.706537
no. of observations        129  no. of parameters          4
AIC.T              -605.413074  AIC                -4.6931246
mean(DLKP)           0.0163011  var(DLKP)         0.000689048
sigma                0.0224149  sigma^2           0.000502426

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
    -0.069181      -0.28445      0.016301
```

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

ARMA(0,1)

```
---- Maximum likelihood estimation of ARFIMA(0,0,1) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                     Coefficient  Std.Error  t-value  t-prob
MA-1                    0.402465    0.07256     5.55   0.000
Constant               0.0163039   0.002902     5.62   0.000

log-likelihood      300.421515
no. of observations        129  no. of parameters          3
AIC.T              -594.843029  AIC                -4.61118627
mean(DLKP)           0.0163011  var(DLKP)          0.000689048
sigma                0.0235537  sigma^2            0.000554777

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.23243      0.016301
```

ARMA(0,0)

```
---- Maximum likelihood estimation of ARFIMA(0,0,0) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                    Coefficient  Std.Error  t-value  t-prob
Constant              0.0163011   0.002311     7.05   0.000

log-likelihood     286.529809
no. of observations        129  no. of parameters          2
AIC.T              -569.059617  AIC              -4.41131486
mean(DLKP)           0.0163011  var(DLKP)        0.000689048
sigma                0.0262497  sigma^2          0.000689048

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     0.016301
```


(5) Model – Progress...

GiveWin gives them all a number. You can identify them by the value given for the maximum of the log-likelihood function. ARFIMA(6) turns out to be the ARMA(1,0) model.

These are the information criteria. Only use the first two. AIC is not consistent and does not punish enough for extra variables, so will often recommend a model that is too large. In this case all three are smallest for the ARMA(1,0) model.

Progress to date

Model	T	p		log-likelihood	SC	HQ	AIC
Arfima(1)	129	5	EML	308.74511	-4.5984	-4.6642	-4.7092
Arfima(2)	129	4	EML	307.19850	-4.6121	-4.6647	-4.7008
Arfima(3)	129	3	EML	307.13887	-4.6488	-4.6883	-4.7153
Arfima(4)	129	4	EML	308.11364	-4.6263	-4.6789	-4.7149
Arfima(5)	129	3	EML	306.97364	-4.6463	-4.6857	-4.7128
Arfima(6)	129	2	EML	306.28278	-4.6732	-4.6995	-4.7176
Arfima(7)	129	3	EML	306.70654	-4.6421	-4.6816	-4.7086
Arfima(8)	129	2	EML	300.42151	-4.5824	-4.6087	-4.6267
Arfima(9)	129	1	EML	286.52981	-4.4046	-4.4178	-4.4268

cont...

cont...

```
Tests of model reduction (please ensure models are nested for test validity)
Arfima( 1) --> Arfima( 2): Chi^2( 1) =      3.0932 [0.0786]
Arfima( 1) --> Arfima( 3): Chi^2( 2) =      3.2125 [0.2006]
Arfima( 1) --> Arfima( 4): Chi^2( 1) =      1.2629 [0.2611]
Arfima( 1) --> Arfima( 5): Chi^2( 2) =      3.5429 [0.1701]
Arfima( 1) --> Arfima( 6): Chi^2( 3) =      4.9246 [0.1774]
Arfima( 1) --> Arfima( 7): Chi^2( 2) =      4.0771 [0.1302]
Arfima( 1) --> Arfima( 8): Chi^2( 3) =      16.647 [0.0008] **
Arfima( 1) --> Arfima( 9): Chi^2( 4) =      44.431 [0.0000] **

Arfima( 2) --> Arfima( 3): Chi^2( 1) =     0.11925 [0.7298]
Arfima( 2) --> Arfima( 5): Chi^2( 1) =     0.44971 [0.5025]
Arfima( 2) --> Arfima( 6): Chi^2( 2) =      1.8314 [0.4002]
Arfima( 2) --> Arfima( 7): Chi^2( 1) =     0.98392 [0.3212]
Arfima( 2) --> Arfima( 8): Chi^2( 2) =      13.554 [0.0011] **
Arfima( 2) --> Arfima( 9): Chi^2( 3) =      41.337 [0.0000] **

Arfima( 3) --> Arfima( 6): Chi^2( 1) =      1.7122 [0.1907]
Arfima( 3) --> Arfima( 8): Chi^2( 1) =      13.435 [0.0002] **
Arfima( 3) --> Arfima( 9): Chi^2( 2) =      41.218 [0.0000] **

Arfima( 4) --> Arfima( 5): Chi^2( 1) =      2.2800 [0.1311]
Arfima( 4) --> Arfima( 6): Chi^2( 2) =      3.6617 [0.1603]
Arfima( 4) --> Arfima( 7): Chi^2( 1) =      2.8142 [0.0934]
Arfima( 4) --> Arfima( 8): Chi^2( 2) =      15.384 [0.0005] **
Arfima( 4) --> Arfima( 9): Chi^2( 3) =      43.168 [0.0000] **

Arfima( 5) --> Arfima( 6): Chi^2( 1) =      1.3817 [0.2398]
Arfima( 5) --> Arfima( 8): Chi^2( 1) =      13.104 [0.0003] **
Arfima( 5) --> Arfima( 9): Chi^2( 2) =      40.888 [0.0000] **

Arfima( 6) --> Arfima( 9): Chi^2( 1) =      39.506 [0.0000] **

Arfima( 7) --> Arfima( 8): Chi^2( 1) =      12.570 [0.0004] **
Arfima( 7) --> Arfima( 9): Chi^2( 2) =      40.353 [0.0000] **

Arfima( 8) --> Arfima( 9): Chi^2( 1) =      27.783 [0.0000] **
```

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

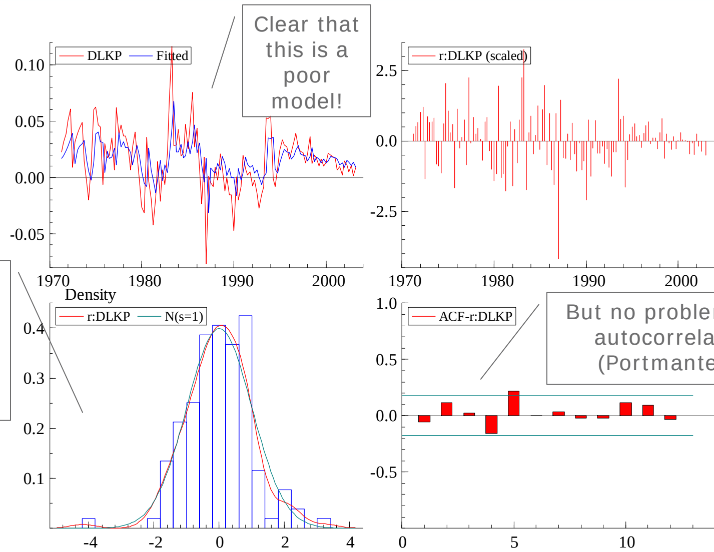
(6) Re-estimate ARMA(1,0); Test – Test Summary

```
---- Maximum likelihood estimation of ARFIMA(1,0,0) model ----
The estimation sample is:  1971 (2) - 2003 (2)
The dependent variable is: DLKP  (TimeSeries.in7)

                  Coefficient  Std.Error  t-value  t-prob
AR-1                 0.511624    0.07500     6.82   0.000
Constant            0.0162915   0.004022     4.05   0.000

log-likelihood     306.282782
no. of observations       129  no. of parameters           3
AIC.T             -606.565565  AIC                -4.70205864
mean(DLKP)          0.0163011  var(DLKP)          0.000689048
sigma               0.0224964  sigma^2            0.000506088

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
      0.51504      0.016301

Descriptive statistics for residuals:
Normality test:    Chi^2(2)  =   20.973 [0.0000]**
ARCH 1-1 test:     F(1,125)  =   3.9632 [0.0487]*
Portmanteau(12):   Chi^2(11)=    15.567 [0.1580]
```

Rejects null of normality for the residuals!

This is a test for whether there is autocorrelation in the residuals. Accepts null of no autocorrelation.

Test – Graphic Analysis



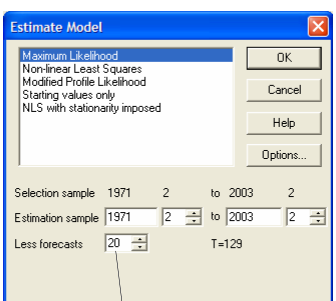
Clear that this is a poor model!

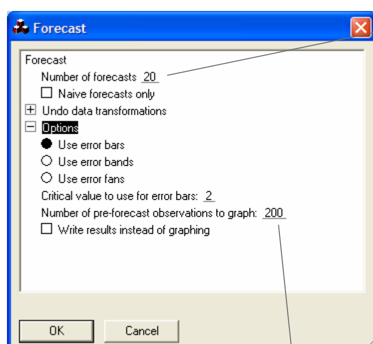
The non-normality of the residuals is clear from this.

But no problem with autocorrelation (Portmanteau)

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

(7) Model – Estimate...; Test – Forecast...





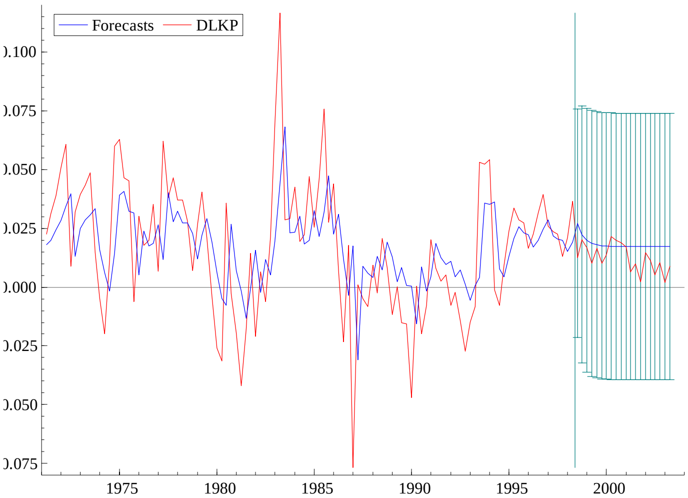
Put this equal to 20.

We will see how well the model forecasts for the last 20 periods.

Remember to change this!

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

The forecasts



With 95% certainty we will stay within the historical range of the variable, minus extremes.

The “problem” is, that we start the forecast close to the historic mean.

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

(8) Try yourselves with a new time series…

… if you have time!

DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

Next time

- We learn how to test for unit roots!