DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

Econometrics 2, Class 1

Problem Set #11
November 28, 2005

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#11.1 Unit roots and cointegration

We will go through this old exam question on the blackboard.

#11.2 Engle-Granger analysis for Danish consumption

In this exercise we want to construct a single equation cointegration model for the Danish private consumption. We define the vector of variables

$$\mathbf{Z}_t = \begin{pmatrix} C_t \\ Y_t \\ W_t \\ \mathsf{ARBLOS}_t \end{pmatrix} = \begin{pmatrix} \log(\mathsf{FCP}_t) \\ \log(\mathsf{FYDP}_t) \\ \log(\mathsf{REALFOR}_t) \\ \mathsf{ARBLOS}_t \end{pmatrix},$$

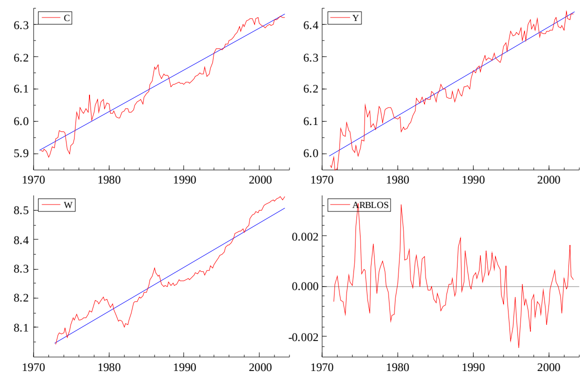
where FCP is real Danish private consumption, FYDP is real disposable income, REALFOR is real wealth including the value of owner occupied housing, and ARBLOS is the income loss from changes in unemployment. The variables are located in `ConsumptionData.In7` introduced in #2.2.

You will need to start by defining C, Y and W in the dataset.

Test for cointegration – 3 steps

1) Check for unit roots in the variables. (Question 1)
2) Estimate the static long-run relation using OLS and save the estimated residuals. (Question 2)
3) Perform the Engle-Granger residual-based test (i.e. ADF on estimated residuals) for whether the long-run relation is a cointegrating relation. (Question 3)

(1) Test for unit roots



A quick look at the graphs reveals that C, Y and W are trended so the relevant test is for trend stationarity.

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The usual ADF tests for a unit root

```
EQ( 1) Modelling DC by OLS (using ConsumptionData.in7)
       The estimation sample is: 1971 (3) to 2003 (2)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
DC_1                -0.192405    0.08807    -2.18   0.031   0.0371
Constant             0.819809     0.3043     2.69   0.008   0.0553
C_1                 -0.138096    0.05154    -2.68   0.008   0.0547
Trend             0.000442016  0.0001726     2.56   0.012   0.0503

sigma                0.0176165  RSS              0.0384821446
R^2                    0.11958  F(3,124) =     5.614 [0.001]**
log-likelihood          337.39  DW                      1.97
no. of observations        128  no. of parameters          4
mean(DC)            0.00325285  var(DC)           0.000341475
EQ( 2) Modelling DY by OLS (using ConsumptionData.in7)
       The estimation sample is: 1971 (3) to 2003 (2)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
DY_1                -0.143814    0.08824    -1.63   0.106   0.0210
Constant              1.63477     0.4159     3.93   0.000   0.1108
Y_1                 -0.272094    0.06947    -3.92   0.000   0.1101
Trend             0.000914763  0.0002455     3.73   0.000   0.1007

sigma                0.0234463  RSS              0.0681664246
R^2                   0.179754  F(3,124) =     9.058 [0.000]**
log-likelihood         300.797  DW                      1.98
no. of observations        128  no. of parameters          4
mean(DY)            0.00372038  var(DY)           0.000649257
```

The 5% critical value in both cases is -3.45.

This means than we cannot reject the null of a unit root for *c*. However, *y* could be stationary. But it seems strange to assume that *y* is stationary and does not cointegrate with *c* (at least according to economic theory).

The usual ADF tests for a unit root (cont.)

EQ(3) Modelling DW by OLS (using ConsumptionData.in7)
The estimation sample is: 1973 (3) to 2003 (2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DW_1	0.0309362	0.09071	0.341	0.734	0.0010
Constant	0.339114	0.2219	1.53	0.129	0.0197
W_1	-0.0420734	0.02769	-1.52	0.131	0.0195
Trend	0.000183417	0.0001079	1.70	0.092	0.0243

sigma	0.0119327	RSS	0.0165171035
R^2	0.0255355	F(3,116) =	1.013 [0.390]
log-likelihood	363.178	DW	2.03
no. of observations	120	no. of parameters	4
mean(DW)	0.00392522	var(DW)	0.000141249

The critical value for the test for a unit root in *w* is -3.45.

We cannot therefore reject the null of a unit root.

EQ(5) Modelling DARBLOS by OLS (using ConsumptionData.in7)
The estimation sample is: 1972 (3) to 2003 (2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DARBLOS_1	0.212978	0.1009	2.11	0.037	0.0361
DARBLOS_2	-0.341076	0.08534	-4.00	0.000	0.1184
DARBLOS_3	0.205151	0.09213	2.23	0.028	0.0400
Constant	3.93807e-005	6.083e-005	0.647	0.519	0.0035
ARBLOS_1	-0.287906	0.07716	-3.73	0.000	0.1047

sigma	0.000671644	RSS	5.36816446e-005
R^2	0.323143	F(4,119) =	14.2 [0.000]**
log-likelihood	732.52	DW	1.97
no. of observations	124	no. of parameters	5
mean(DARBLOS)	4.68274e-006	var(DARBLOS)	6.39598e-007

The critical value for the test for a unit root in *arblos* is

-2.89. We thus reject the null of a unit root.

(2) OLS

Estimate this static long-run relation:

$$C_t = \beta_0 + \beta_1 Y_t + \beta_2 W_t + u_t,$$

EQ(1) Modelling C by OLS (using ConsumptionData.in7)
The estimation sample is: 1973 (1) to 2003 (2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	-0.403624	0.1290	-3.13	0.002	0.0761
Y	0.363714	0.04920	7.39	0.000	0.3147
W	0.516203	0.04418	11.7	0.000	0.5343

sigma	0.0227115	RSS	0.0613817803
R^2	0.963165	F(2,119) =	1556 [0.000]**
log-likelihood	290.164	DW	0.574
no. of observations	122	no. of parameters	3
mean(C)	6.13522	var(C)	0.0136591

Remember: the estimates are super consistent, but we cannot use e.g. the t-values.

There is a less than one-to-one increase in consumption associated with an increase in income or wealth.

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(3) Engle-Granger residual based test for cointegration

This is just an ADF test for a unit root on the estimated residuals.

- Start by saving the estimated residuals.
- Then perform the usual ADF test on the estimated residuals. Remember that you should not use a constant, since the residuals have mean 0.

$$\Delta\widehat{u}_t = \sum_{i=1}^{k-1} c_i \Delta\widehat{u}_{t-i} + \pi\widehat{u}_{t-1} + \eta_t.$$

Here just one lag has been chosen.

```
EQ( 1) Modelling DResNoTrend by OLS (using ConsumptionData.in7)
       The estimation sample is: 1973 (3) to 2003 (2)

                Coefficient  Std.Error  t-value  t-prob Part.R^2
DResNoTrend_1     -0.223486    0.08882    -2.52   0.013   0.0509
ResNoTrend_1      -0.221118    0.06759    -3.27   0.001   0.0832

sigma             0.0153836  RSS              0.0279252315
log-likelihood       331.67  DW                       1.93
no. of observations     120  no. of parameters           2
mean(DResNoTrend) 6.32423e-005  var(DResNoTrend)  0.000286783
```

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Engle-Granger residual based test for cointegration (cont.)

(A) Residual-based (ADF) test for no-cointegration

Number of estimated parameters	Constant in (22)			Constant and trend in (22)		
	1%	5%	10%	1%	5%	10%
0	−3.43	−2.86	−2.57	−3.96	−3.41	−3.13
1	−3.90	−3.34	−3.04	−4.32	−3.78	−3.50
2	−4.29	−3.74	−3.45	−4.66	−4.12	−3.84
3	−4.64	−4.10	−3.81	−4.97	−4.43	−4.15
4	−4.96	−4.42	−4.13	−5.25	−4.72	−4.43

The critical values depend on the *static* regression, where there is a constant and two explanatory variables.

The t-value of -3.27 means that we cannot reject the null of a unit root / no-cointegration.

Engle-Granger residual based test for cointegration (cont.)

It might also be worth checking whether it would have made a difference adding a trend to the static relation…

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	0.275007	0.5178	0.531	0.596	0.0024
Y	0.302734	0.06660	4.55	0.000	0.1490
W	0.476997	0.05270	9.05	0.000	0.4097
Trend	0.000371416	0.0002745	1.35	0.179	0.0153

sigma	0.0226327	RSS	0.0604441512
R^2	0.963728	F(3,118) =	1045 [0.000]**
log-likelihood	291.103	DW	0.564
no. of observations	122	no. of parameters	4
mean(C)	6.13522	var(C)	0.0136591

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DResTrend_1	-0.214996	0.08933	-2.41	0.018	0.0468
ResTrend_1	-0.218484	0.06739	-3.24	0.002	0.0818

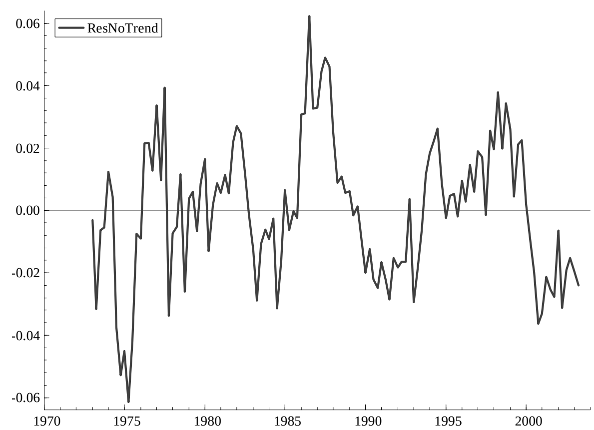
sigma	0.015245	RSS	0.027424364
log-likelihood	332.756	DW	1.93
no. of observations	120	no. of parameters	2
mean(DResTrend)	1.6543e-005	var(DResTrend)	0.000278973

… but the trend is insignificant, and the relevant t-value is almost identical. We have removed the stochastic *and* the deterministic trend (if there was one).

(4) Define the error correction term

We assume that the variables cointegrate anyway!
We can then define
$ecm_t = \widehat{u}_t$

There are some large deviations, which suggest that the process is still non-stationary. (Hence the rejection of cointegration.)



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(5) Single equation ECM for *c*

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta C_{t-1} + \alpha_2 \Delta Y_t + \alpha_3 \Delta Y_{t-1} + \alpha_4 \Delta W_t + \alpha_5 \Delta W_{t-1}$$
$$+ \alpha_6 \text{ARBLOS}_t + \alpha_7 \text{ecm}_{t-1} + \epsilon_t.$$

```
EQ( 7) Modelling DC by OLS (using ConsumptionData.in7)
       The estimation sample is: 1973 (3) to 2003 (2)
```

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DC_1	-0.222952	0.09099	-2.45	0.016	0.0509
Constant	0.00239323	0.001564	1.53	0.129	0.0205
DY	0.225494	0.06034	3.74	0.000	0.1109
DY_1	-0.0173814	0.06355	-0.273	0.785	0.0007
DW	0.404121	0.1173	3.44	0.001	0.0958
DW_1	-0.00261191	0.1273	-0.0205	0.984	0.0000
ARBLOS	-4.83216	1.434	-3.37	0.001	0.0920
ResNoTrend_1	-0.298474	0.06861	-4.35	0.000	0.1446

sigma	0.0146984	RSS	0.0241968338
R^2	0.422509	F(7,112) =	11.71 [0.000]**
log-likelihood	340.269	DW	1.94
no. of observations	120	no. of parameters	8
mean(DC)	0.00310834	var(DC)	0.000349166

There is strong evidence here that *c* error corrects.

The coefficient implies that 30% of deviations from equilibrium are removed each quarter (which seems like a lot!).

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Single equation ECM for *y*

```
EQ( 8) Modelling DY by OLS (using ConsumptionData.in7)
       The estimation sample is: 1973 (3) to 2003 (2)
```

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DY_1	-0.349647	0.08786	-3.98	0.000	0.1239
Constant	0.00193399	0.002326	0.831	0.408	0.0061
DC	0.491604	0.1316	3.74	0.000	0.1109
DC_1	0.426863	0.1319	3.24	0.002	0.0856
DW	-0.126527	0.1818	-0.696	0.488	0.0043
DW_1	-0.154450	0.1874	-0.824	0.412	0.0060
ARBLOS	1.58824	2.218	0.716	0.475	0.0046
ResNoTrend_1	0.0675447	0.1093	0.618	0.538	0.0034

sigma	0.0217026	RSS	0.0527521477
R^2	0.253263	F(7,112) =	5.427 [0.000]**
log-likelihood	293.506	DW	1.97
no. of observations	120	no. of parameters	8
mean(DY)	0.00280133	var(DY)	0.000588697

No evidence of error correction here...

Single equation ECM for *w*

EQ(9) Modelling DW by OLS (using ConsumptionData.in7)
The estimation sample is: 1973 (3) to 2003 (2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DW_1	0.0225024	0.09747	0.231	0.818	0.0005
Constant	0.00334798	0.001168	2.87	0.005	0.0683
DC	0.236979	0.06881	3.44	0.001	0.0958
DC_1	-0.0506646	0.07136	-0.710	0.479	0.0045
DY	-0.0340330	0.04890	-0.696	0.488	0.0043
DY_1	0.0149141	0.04866	0.306	0.760	0.0008
ARBLOS	-0.299853	1.152	-0.260	0.795	0.0006
ResNoTrend_1	-0.0141432	0.05679	-0.249	0.804	0.0006

sigma	0.0112556	RSS	0.0141891796
R^2	0.162877	F(7,112) =	3.113 [0.005]**
log-likelihood	372.293	DW	2.01
no. of observations	120	no. of parameters	8
mean(DW)	0.00392522	var(DW)	0.000141249

... or here!

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(6) Select the preferred Engle-Granger model for *dc*

EQ(10) Modelling DC by OLS (using ConsumptionData.in7)
The estimation sample is: 1973 (3) to 2003 (2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DC_1	-0.222952	0.09099	-2.45	0.016	0.0509
Constant	0.00239323	0.001564	1.53	0.129	0.0205
DY	0.225494	0.06034	3.74	0.000	0.1109
DY_1	-0.0173814	0.06355	-0.273	0.785	0.0007
DW	0.404121	0.1173	3.44	0.001	0.0958
DW_1	-0.00261191	0.1273	-0.0205	0.984	0.0000
ARBLOS	-4.83216	1.434	-3.37	0.001	0.0920
ResNoTrend_1	-0.298474	0.06861	-4.35	0.000	0.1446

sigma	0.0146984	RSS	0.0241968338
R^2	0.422509	F(7,112) =	11.71 [0.000]**
log-likelihood	340.269	DW	1.94
no. of observations	120	no. of parameters	8
mean(DC)	0.00310834	var(DC)	0.000349166

dy_1 and *dw_1* are insignificant. Do not delete the constant!

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Select the preferred Engle-Granger model for *dc* (cont.)

EQ(11) Modelling DC by OLS (using ConsumptionData.in7)
The estimation sample is: 1973 (2) to 2003 (2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DC_1	-0.243964	0.07492	-3.26	0.001	0.0844
Constant	0.00239308	0.001472	1.63	0.107	0.0225
DY	0.224344	0.05492	4.09	0.000	0.1267
DW	0.372732	0.1131	3.30	0.001	0.0863
ARBLOS	-4.85374	1.408	-3.45	0.001	0.0937
ResNoTrend_1	-0.294183	0.06296	-4.67	0.000	0.1596

sigma	0.0145894	RSS	0.0244776669
R^2	0.415835	F(5,115) =	16.37 [0.000]**
log-likelihood	342.908	DW	1.94
no. of observations	121	no. of parameters	6
mean(DC)	0.00312004	var(DC)	0.000346297

AR 1-5 test:	F(5,110) =	0.71827	[0.6111]
ARCH 1-4 test:	F(4,107) =	0.97420	[0.4249]
Normality test:	Chi^2(2) =	8.8034	[0.0123]*
hetero test:	F(10,104)=	3.1641	[0.0014]**
hetero-X test:	F(20,94) =	2.4689	[0.0019]**
RESET test:	F(1,114) =	0.072152	[0.7887]

It is of course important that we do not have a problem with AR, but this model does not appear well specified.

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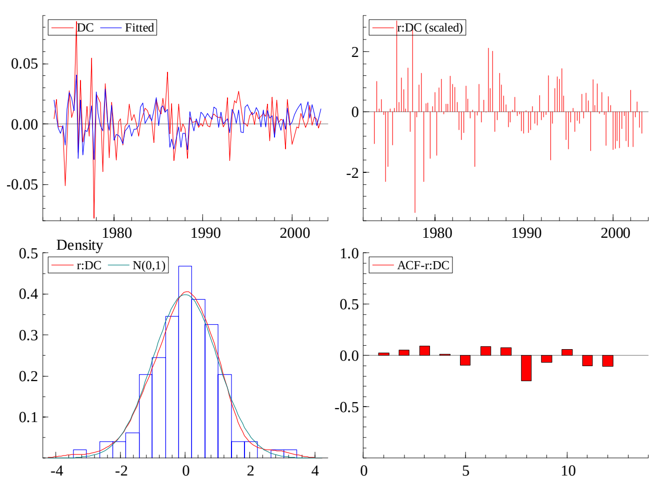
Select the preferred Engle-Granger model for *dc* (cont.)



There are too many large outliers! Especially in 75:4, 77:3 and 77:4.

Normality of the residuals is important because our maximum likelihood based estimation is based on the assumption of normally distributed error terms.

(7) Add dummies for large residuals

- It is sometimes a good idea to "dummy out" extreme observations.
- Adding a dummy for one observation means that that observation is effectively ignored for estimation.
- The residual becomes zero.
- It is, however, always a good idea to have a good reason for using this method. Here we justify it by the fact that the observations are related to two transitory policy measures.
- We use the following code in algebra editor:
  - DUM754=dummy(1975,4,1975,4);
  - DUM773=dummy(1977,3,1977,3);
  - DUM774=dummy(1977,4,1977,4);
- The dummies take the value of one for the specified observation, and 0 otherwise.

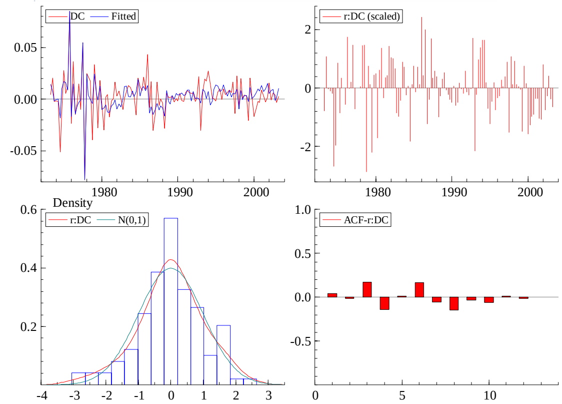
The results

```
EQ(13) Modelling DC by OLS (using ConsumptionData.in7)
       The estimation sample is: 1973 (2) to 2003 (2)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
DC_1                -0.168778    0.06575    -2.57   0.012   0.0556
Constant           0.00229926   0.001246     1.84   0.068   0.0295
DY                  0.0797525    0.05231     1.52   0.130   0.0203
DW                   0.327539    0.09611     3.41   0.001   0.0940
ARBLOS               -4.64170      1.205    -3.85   0.000   0.1169
ResNoTrend_1        -0.230559    0.05479    -4.21   0.000   0.1365
DUM754              0.0628887    0.01399     4.50   0.000   0.1529
DUM773              0.0475595    0.01275     3.73   0.000   0.1105
DUM774             -0.0569334    0.01305    -4.36   0.000   0.1452

sigma                0.012336  RSS             0.0170439211
R^2                  0.593243  F(8,112) =     20.42 [0.000]**
log-likelihood        364.807  DW                       1.91
no. of observations       121  no. of parameters           9
mean(DC)           0.00312004  var(DC)           0.000346297
```

Note that *dy* has now become insignificant. Its significance in the previous model was entirely due to the fact that outliers in *y* occurred simultaneously with those in *c*.

Specification tests

```
AR 1-5 test:       F(5,107) =   1.8316 [0.1128]
ARCH 1-4 test:     F(4,104) =  0.68248 [0.6056]
Normality test:    Chi^2(2) =   2.9693 [0.2266]
hetero test:       F(13,98) =   1.1039 [0.3650]
hetero-X test:     F(23,88) =  0.71810 [0.8152]
RESET test:        F(1,111) =  0.88247 [0.3496]
```



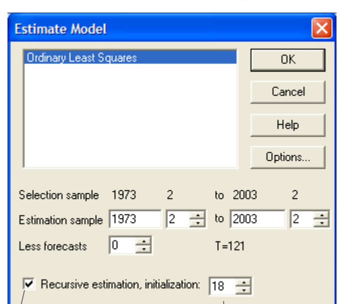
This model looks amazingly well specified!

Inference is now standard, since all unit roots have been removed.

(8) Recursive estimation

This simply involves estimating all the coefficients for all sample lengths up to *T*.

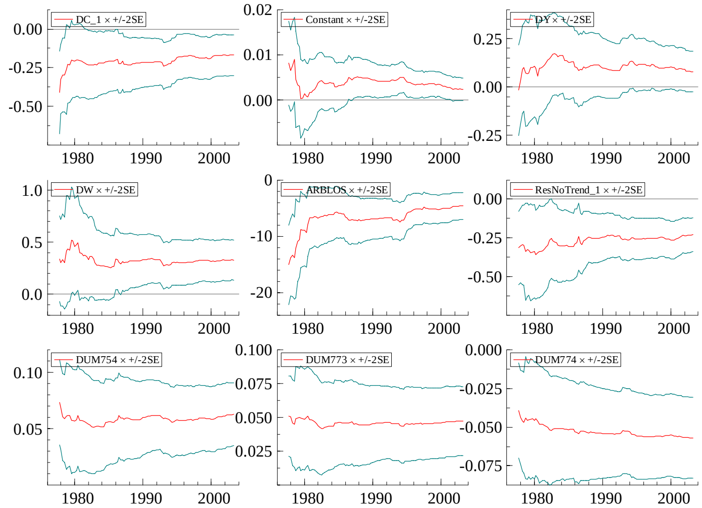
Since the model is estimated under the assumption of constant coefficients, the estimators should not fluctuate too much.



Remember to check this box before running the estimation. Then you can choose "Recursive Graphics" in the Test menu.

This should be set equal to twice the number of coefficients to be estimated. It specifies the smallest sample length to be used.

The result



A rule of thumb is that no estimate should be within the last confidence bands.

This seems to be fairly well achieved.

Note that confidence bands should narrow, since our estimators are consistent (except for the dummies, which are not).

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#11.3 ADL analysis for Danish consumption

(1) An alternative to the (static) Engle-Granger approach is cointegration analysis based on an unrestricted ADL or ECM (dynamic) model.

The advantage is that the dynamic model is well specified (i.e. does not ignore dynamic effects), so, *given cointegration*, t-ratios constructed from the estimated standard errors follow standard normal distributions under the null.

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(2) Estimate the unrestricted ADL model

$$\begin{aligned} C_t &= \beta_0 + \beta_1 C_{t-1} + \beta_2 C_{t-2} + \beta_3 Y_t + \beta_4 Y_{t-1} + \beta_5 Y_{t-2} \\ &+ \beta_6 W_t + \beta_7 W_{t-1} + \beta_8 W_{t-2} + \beta_9 \text{ARBLOS}_t \\ &+ \beta_{10} Dum754_t + \beta_{11} Dum773_t + \beta_{12} Dum774_t + \epsilon_t. \end{aligned}$$

```
EQ(16) Modelling C by OLS (using ConsumptionData.in7)
       The estimation sample is: 1973 (3) to 2003 (2)
```

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
C_1	0.597538	0.08330	7.17	0.000	0.3248
C_2	0.162208	0.07952	2.04	0.044	0.0374
Constant	0.0344309	0.08075	0.426	0.671	0.0017
Y	0.0741451	0.05829	1.27	0.206	0.0149
Y_1	-0.0205322	0.06180	-0.332	0.740	0.0010
Y_2	0.0579263	0.05815	0.996	0.321	0.0092
W	0.321061	0.1044	3.07	0.003	0.0812
W_1	-0.178497	0.1490	-1.20	0.233	0.0132
W_2	-0.0523076	0.1105	-0.473	0.637	0.0021
ARBLOS	-5.17642	1.356	-3.82	0.000	0.1198
DUM754	0.0631710	0.01467	4.31	0.000	0.1478
DUM773	0.0476979	0.01329	3.59	0.001	0.1074
DUM774	-0.0564696	0.01320	-4.28	0.000	0.1460

sigma	0.0123577	RSS	0.0163403029
R^2	0.989757	F(12,107) =	861.6 [0.000]**
log-likelihood	363.824	DW	1.9
no. of observations	120	no. of parameters	13
mean(C)	6.13834	var(C)	0.0132934

Here you could try deleting insignificant terms.

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(3) Misspecification tests

```
AR 1-5 test:      F(5,102) =   1.9421 [0.0938]
ARCH 1-4 test:    F(4,99)  =  0.90698 [0.4630]
Normality test:   Chi^2(2) =   4.5833 [0.1011]
hetero test:      F(21,85) =  0.82281 [0.6851]
Hetero-X test: not enough observations
RESET test:       F(1,106) =   3.1797 [0.0774]
```

The model appears to be well specified.

The Hetro-X test involves cross terms for all coefficients etc., so with this number of parameters it is not possible to give a test statistic.

(4) Test – Dynamic Analysis – Static long-run solution

```
Solved static long-run equation for C
                    Coefficient  Std.Error  t-value  t-prob
Constant               0.143311     0.3461    0.414   0.680
Y                      0.464256     0.1320     3.52   0.001
W                      0.375674     0.1170     3.21   0.002
ARBLOS                 -21.5456      6.822    -3.16   0.002
DUM754                 0.262935    0.09940     2.65   0.009
DUM773                 0.198531    0.06970     2.85   0.005
DUM774                -0.235042    0.08721    -2.70   0.008
Long-run sigma = 0.0514361

ECM = C - 0.143311 - 0.464256*Y - 0.375674*W + 21.5456*ARBLOS
      - 0.262935*DUM754 - 0.198531*DUM773 + 0.235042*DUM774;
WALD test: Chi^2(6) = 588.127 [0.0000] **
```

$$\frac{\hat{\beta}_3+\hat{\beta}_4+\hat{\beta}_5}{1-\hat{\beta}_1-\hat{\beta}_2}=\frac{0.07-0.02+0.06}{1-0.60-0.16}=0.46$$

This is a test of the null that all parameters are equal to 0 (rejected).

(5) Test – Dynamic Analysis – Lag structure analysis (PcGive test for no-cointegration)

```
Tests on the significance of each variable
Variable     F-test      Value [  Prob]    Unit-root t-test
C            F(2,107) =  80.264 [0.0000]**      -3.9539
Constant     F(1,107) = 0.18181 [0.6707]
Y            F(3,107) =  3.1883 [0.0267]*        2.862
W            F(3,107) =  4.9486 [0.0030]**       2.2353
ARBLOS       F(1,107) =  14.565 [0.0002]**      -3.8164
DUM754       F(1,107) =  18.555 [0.0000]**       4.3076
DUM773       F(1,107) =  12.878 [0.0005]**       3.5886
DUM774       F(1,107) =  18.299 [0.0000]**      -4.2777
```

Don't use these!

We thus reject the null of no-cointegration.

(B) PcGive test for no-cointegration

Number of variables in X_t (p)	Constant in (25)			Constant and trend in (25)		
	1%	5%	10%	1%	5%	10%
2	−3.79	−3.21	−2.91	−4.25	−3.69	−3.39
3	−4.09	−3.51	−3.19	−4.50	−3.93	−3.62
4	−4.36	−3.76	−3.44	−4.72	−4.14	−3.83
5	−4.59	−3.99	−3.66	−4.93	−4.34	−4.03

Table 1: Asymptotic critical values for tests of no-cointegration. Reproduced from Davidson and MacKinnon (1993).

(6) Repetition from the lectures: Dynamic multipliers

- From the equations

$$\begin{aligned} Y_t &= \delta + \theta Y_{t-1} + \phi_0 X_t + \phi_1 X_{t-1} + \epsilon_t \\ Y_{t+1} &= \delta + \theta Y_t + \phi_0 X_{t+1} + \phi_1 X_t + \epsilon_{t+1} \\ Y_{t+2} &= \delta + \theta Y_{t+1} + \phi_0 X_{t+2} + \phi_1 X_{t+1} + \epsilon_{t+2} \end{aligned}$$

we can find the dynamic multipliers as the derivatives:

$$\begin{aligned} \frac{\partial Y_t}{\partial X_t} &= && && \phi_0 \\ \frac{\partial Y_{t+1}}{\partial X_t} &= \frac{\partial Y_t}{\partial X_{t-1}} &&= \theta \frac{\partial Y_t}{\partial X_t} + \phi_1 &&= \theta\phi_0 + \phi_1 \\ \frac{\partial Y_{t+2}}{\partial X_t} &= \frac{\partial Y_t}{\partial X_{t-2}} &&= \theta \frac{\partial Y_{t+1}}{\partial X_t} &&= \theta\left(\theta\phi_0 + \phi_1\right) \\ \frac{\partial Y_{t+3}}{\partial X_t} &= \frac{\partial Y_t}{\partial X_{t-3}} &&= \theta \frac{\partial Y_{t+2}}{\partial X_t} &&= \theta^2\left(\theta\phi_0 + \phi_1\right) \\ &\vdots \\ \frac{\partial Y_{t+k}}{\partial X_t} &= \frac{\partial Y_t}{\partial X_{t-k}} &&= &&\theta^{k-1}\left(\theta\phi_0 + \phi_1\right). \end{aligned}$$

Due to stationarity, $|\theta| < 1$, shocks have transitory effects:

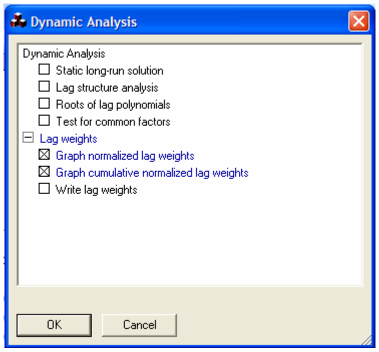
$$\frac{\partial Y_{t+k}}{\partial X_t} \to 0 \quad \text{as} \quad k \to \infty.$$

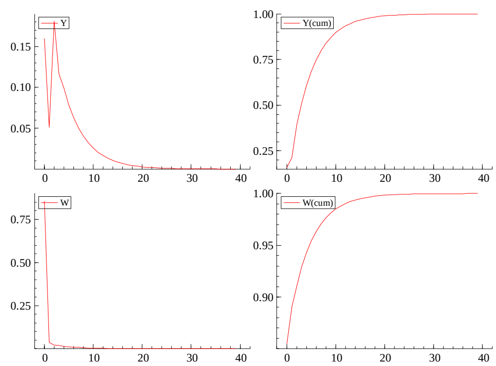
Repetition from the lectures: Dynamic multipliers (cont.)

- Now consider a permanent shift in X_t, so that $E[X_t]$ is changed.
  The final effect in Y_t is the long-run multiplier, given by the accumulated effect

$$\begin{aligned} \frac{\partial Y_t}{\partial X_t} + \frac{\partial Y_t}{\partial X_{t-1}} + \frac{\partial Y_t}{\partial X_{t-2}} + \ldots &= \frac{\partial Y_t}{\partial X_t} + \frac{\partial Y_{t+1}}{\partial X_t} + \frac{\partial Y_{t+2}}{\partial X_t} + \ldots \\ &= \phi_0 + \left(\theta\phi_0 + \phi_1\right) + \theta\left(\theta\phi_0 + \phi_1\right) + \theta^2\left(\theta\phi_0 + \phi_1\right) + \ldots \\ &= \phi_0\left(1 + \theta + \theta^2 + \ldots\right) + \phi_1\left(1 + \theta + \theta^2 + \ldots\right) \\ &= \frac{\phi_0 + \phi_1}{1 - \theta}. \end{aligned}$$

(6) Test – Dynamic Analysis





Note that PcGive normalizes the long-run multipliers (which are the sum of the dynamic multipliers) to one.

Almost the entire impact of a change in *w* comes in the first period.

(7) Unrestricted error correction model

$$\begin{aligned}\Delta C_t &= \alpha_0 + \alpha_1 \Delta C_{t-1} + \alpha_2 \Delta Y_t + \alpha_3 \Delta Y_{t-1} + \alpha_4 \Delta W_t + \alpha_5 \Delta W_{t-1} \\ &+ \alpha_6 C_{t-1} + \alpha_7 Y_{t-1} + \alpha_8 W_{t-1} + \alpha_9 \text{ARBLOS}_t \\ &+ \alpha_{10} Dum754_t + \alpha_{11} Dum773_t + \alpha_{12} Dum774_t + \epsilon_t.\end{aligned}$$

```
EQ(18) Modelling DC by OLS (using ConsumptionData.in7)
       The estimation sample is: 1973 (3) to 2003 (2)
```

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DC_1	-0.162208	0.07952	-2.04	0.044	0.0374
Constant	0.0344309	0.08075	0.426	0.671	0.0017
DY	0.0741451	0.05829	1.27	0.206	0.0149
DY_1	-0.0579263	0.05815	-0.996	0.321	0.0092
DW	0.321061	0.1044	3.07	0.003	0.0812
DW_1	0.0523076	0.1105	0.473	0.637	0.0021
C_1	-0.240254	0.06076	-3.95	0.000	0.1275
Y_1	0.111539	0.03897	2.86	0.005	0.0711
W_1	0.0902570	0.04038	2.24	0.027	0.0446
ARBLOS	-5.17642	1.356	-3.82	0.000	0.1198
DUM754	0.0631710	0.01467	4.31	0.000	0.1478
DUM773	0.0476979	0.01329	3.59	0.001	0.1074
DUM774	-0.0564696	0.01320	-4.28	0.000	0.1460

sigma	0.0123577	RSS	0.0163403029
R^2	0.610016	F(12,107) =	13.95 [0.000]**
log-likelihood	363.824	DW	1.9
no. of observations	120	no. of parameters	13
mean(DC)	0.00310834	var(DC)	0.000349166

HOMEWORK!

- Derive the long-run solution and compare with the ADL model.
- Perform PcGive test for no-cointegration.
- E-mail me if you have difficulties with this.
- I will expect someone to present their answer next week.

Next time

We look at the ARCH model.